

# A NUMERICAL STUDY OF THE IMPACT OF BEND CURVATURES TO FLOW PATTERNS IN NATURAL CONVECTION LOOPS

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## 1. ABSTRACT

Natural convection loops are used in passive heat exchange systems in many applications, such as nuclear reactor cooling and photovoltaic cells. The fluid flow patterns observed in loops exhibit complex nature in certain geometric configurations even for moderate values of the Rayleigh number. We report the results obtained from 2D unsteady laminar numerical simulations on the impact of bend curvatures on fluid flow patterns in convective loops with the horizontal heater and horizontal cooler (HHHC) configuration.

## 2. INTRODUCTION

The natural convection flows are modelled by the Boussinesq approximation of the Navier-Stokes equations [1, Chapter 11]. Following the non-dimensionalisation introduced in [2] the Boussinesq system for the unknown fluid velocity  $\mathbf{u}$ , pressure  $p$ , and temperature  $T$  reads:

$$\frac{\partial \mathbf{u}}{\partial t} - \epsilon_u \Delta \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \mathbf{g}T = 0 \quad \text{in } W = \Omega \times [0, \tau] \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega \quad (2)$$

$$\frac{\partial T}{\partial t} - \epsilon_T \Delta T + \mathbf{u} \cdot \nabla T = 0 \quad \text{in } W \quad (3)$$

The viscosity parameters in (1) and (3) are defined as  $\epsilon_u = \sqrt{\frac{Pr}{Ra}}$  and  $\epsilon_T = \frac{1}{\sqrt{Pr \cdot Ra}}$ , where  $Ra = \frac{g\beta\Delta T D^3}{\mu\alpha}$  is the Rayleigh number and  $Pr = \frac{\mu}{\alpha}$  is the Prandtl number. In previous definitions  $\mu$  is the molecular diffusivity,  $\alpha$  is the thermal diffusion coefficient,  $\beta$  the fluid expansion constant,  $\Delta T = T_h - T_c$  is the temperature difference between the heater and the cooler, and  $D$  is the characteristic length. The boundary conditions for the velocity are:

$$\mathbf{u} = \mathbf{0} \quad \text{on } \partial\Omega \times [0, \tau] \quad (4)$$

while for temperature we have the adiabatic condition on all walls:

$$\epsilon_T \nabla T \cdot \hat{\mathbf{n}} = 0 \quad \text{on } \partial\Omega_N^T \times [0, \tau] \quad (5)$$

except on the hot ( $\partial\Omega_D^h$ ) and the cold ( $\partial\Omega_D^c$ ) section where we impose inhomogeneous Dirichlet boundary conditions

$$T_h = \frac{1}{2}(1 - e^{-\gamma t}) \quad \text{on } \partial\Omega_D^h \times [0, \tau] \quad \text{and} \quad T_c = -\frac{1}{2}(1 - e^{-\gamma t}) \quad \text{on } \partial\Omega_D^c \times [0, \tau] \quad (6)$$

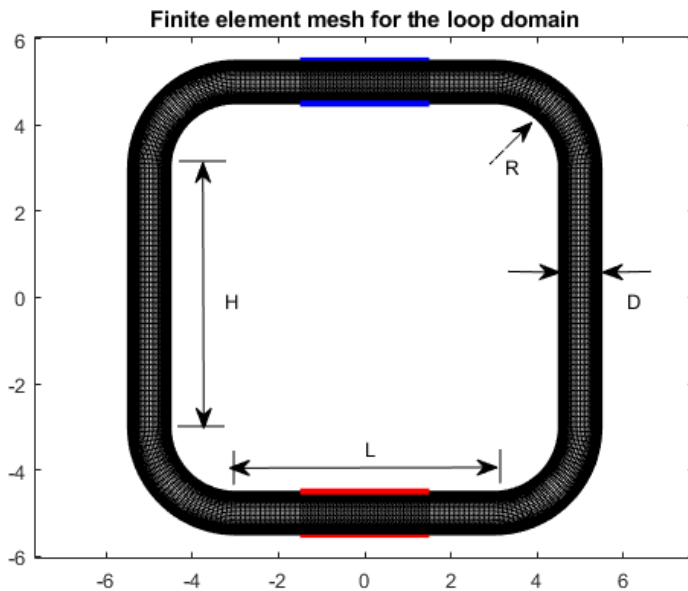
with  $\gamma = 10$ . The exponential term in (6) ensures gradual heating and cooling of the hot and cold segment.

### 3. METHODOLOGY

The problem (1)-(6) is solved approximately in 2D using finite element method. The methodology is described in detail in [2]. The loop domain  $\Omega \cup \partial\Omega$  is discretised by a fixed grid of quadrilateral elements, which are stretched towards the loop walls. The velocity-pressure pair is approximated by the Taylor-Hood method ( $Q_2 - Q_1$ ) [1, p. 133] and the temperature by the biquadratic ( $Q_2$ ) approximation. The resulting discrete non-linear problem is either linearised using the second-order accurate backward difference approximation or solved approximately by the Picard method with a fixed number of steps. The resulting linear system(s) are solved using a direct sparse solver. Discretisation in time is done by the stabilised trapezoidal rule with adaptive step sizes. The adaptivity is achieved through a predictor-corrector scheme using the AB2 method as a predictor. This methodology is implemented in IFISS software library [3].

### 4. RESULTS

We study the impact of bend curvatures to the flow characteristics in natural convection loops. For this purpose, we consider the loop geometry depicted in Fig. 1, which has previously been used in several computer simulations [4]. The difference between the current and previous studies is in the position of the hot and the cold segment, which are now placed on the horizontal branches of the loop (the HHHC configuration). Due to its inherent instability, such configuration is more challenging to simulate than the configurations where the hot/cold segments are placed on the vertical branches of the loop.

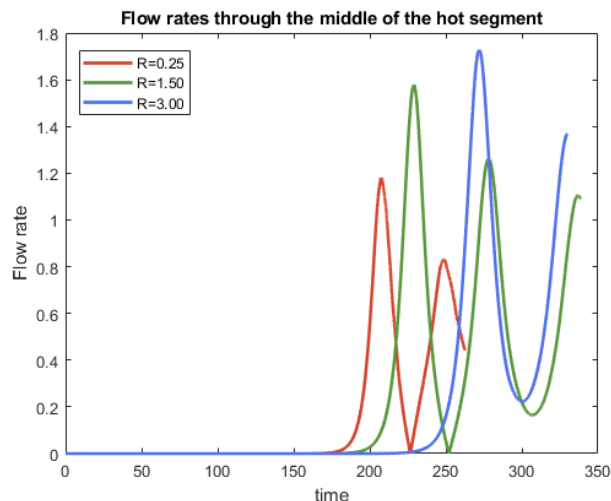


**Fig. 1** The loop domain geometry with  $L = H = 6, D = 1, R = 1.5$ .

The dimensions of the straight segments are  $L = H = 6$ , and the channel width is  $D = 1$ . Both the hot and the cold segment have length 3 and are put symmetrically at the bottom/top branch. We consider three different values for the bend radius, namely  $R = 0.25, 1.5$ , and 3. The fluid parameters are kept constant, with  $Ra = 5 \cdot 10^5$  and  $Pr = 0.71$ . The spatial discretisation is performed with a stretched grid consisting of 60 elements across the channel, and between 44,400 and 49,200 elements in total. This resolution results in discrete problems with 582,280 to 645,340 degrees of

freedom. The non-linear problems are solved using one Picard iteration. The simulations are run over 10,000 time steps with the time accuracy tolerance  $\varepsilon_t = 10^{-5}$ , resulting in the time step sizes in the interval  $[10^{-3}, 10^{-2}]$ .

In Fig. 2 we report the absolute values of the volumetric flow rates for three values of  $R$  calculated across the channel in the middle of the hot segment. The absolute volumetric flow rates are defined as the modulus of the integral of  $v_x$  across the channel. The reason for reporting the absolute flow rates is that the initial flow direction is random (and even depends on the mesh resolution), making the comparisons difficult.



**Fig 2** The moduli of the volumetric flow rates through the middle of the hot segment in loops with three different values of the bend radius  $R$ .

The bend curvature has a notable impact on fluid flow through the loop, both in terms of the local features and the global flow quantities of interest. There are several observations that follow from Fig. 2. There is a long initial phase where no circulation along the loop is observed (the channel is “clogged” with strong vortices). This initial phase is the shortest in cavities with the smallest bend radius, i.e., they appear to be the least stable. When the deadlock is broken, there is a flow surge, with the peak flow rates growing with  $R$ . The flow surge makes the hot fluid going too fast through the cooling segment without being cooled sufficiently. Such hot fluid fills the vertical branch and returns down to the hot segment, creating the buoyancy force that is opposite to the current flow direction. The buoyancy force slows down the flow, or in some cases leads to its reversal. The reversal happens in cases where the flow rate hits 0 in Fig. 2, i.e., for  $R = 0.25$  and  $R = 1.5$ , but not for  $R = 3$ . In all cases the amplitudes of flow rates are reduced with time, but we are unable to draw conclusions about the flow rate asymptotic from the present simulations.

## 5. CONCLUSIONS

We performed a numerical study of the impact of the bend curvature in 2D convection loops to the global flow rates. The simulations show that this geometry parameter has a significant impact on flow, both in terms of the local features and on global quantities such as the transitory and asymptotic flow rates. These effects will be enforced further in the presentation by graphs and flow animations.

## REFERENCES

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